EXAMPLE: SORTABLE ELEMENTS, cl_c AND nc_c
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NATHAN READING

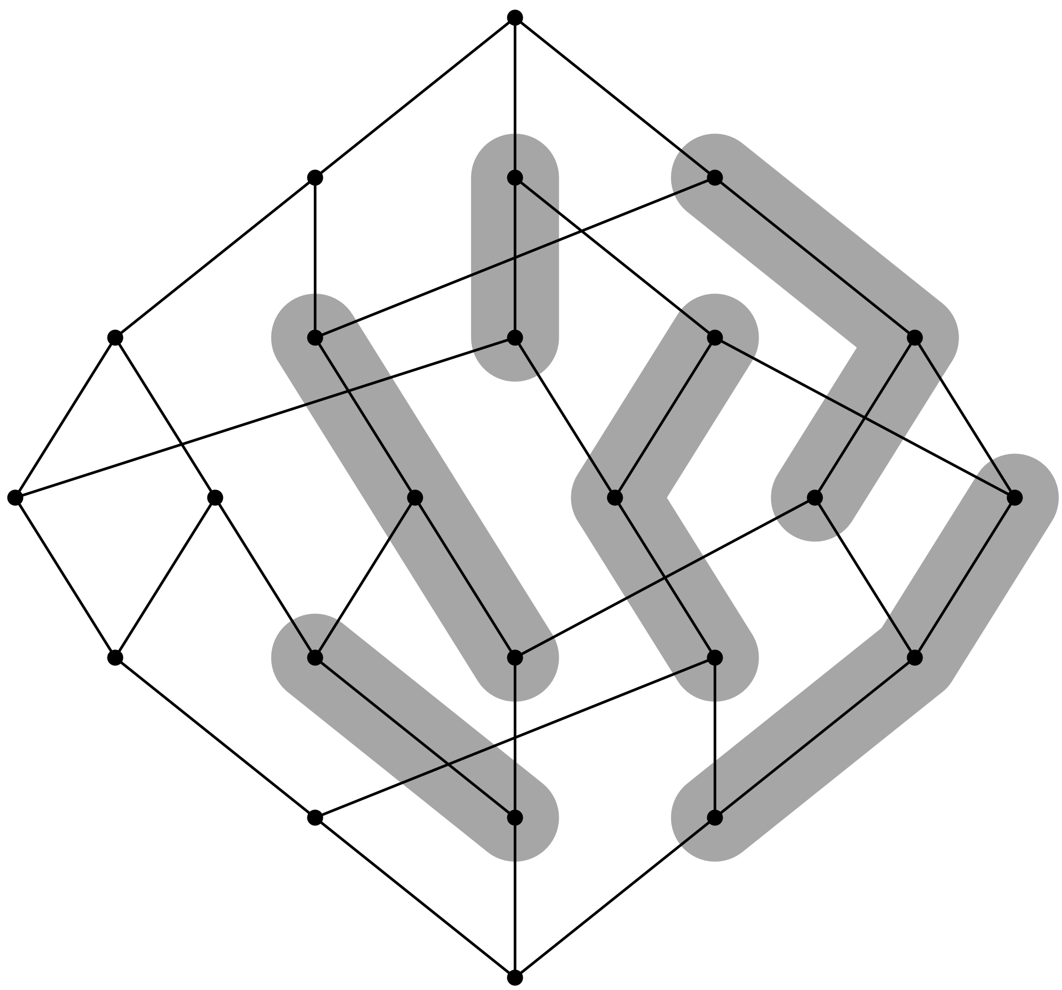

FIGURE 1. The weak order on W of type A_3 showing the c-Cambrian congruence relation. The non-singleton congruence classes are indicated by shading, and non-shaded points are singleton congruence classes. The bottom element is the identity 1 and the elements covering 1 are the elements of S. Naming these (from left to right in the picture) r, s and t, we have $c = rst$, corresponding to an oriented diagram $r \to s \to t$.

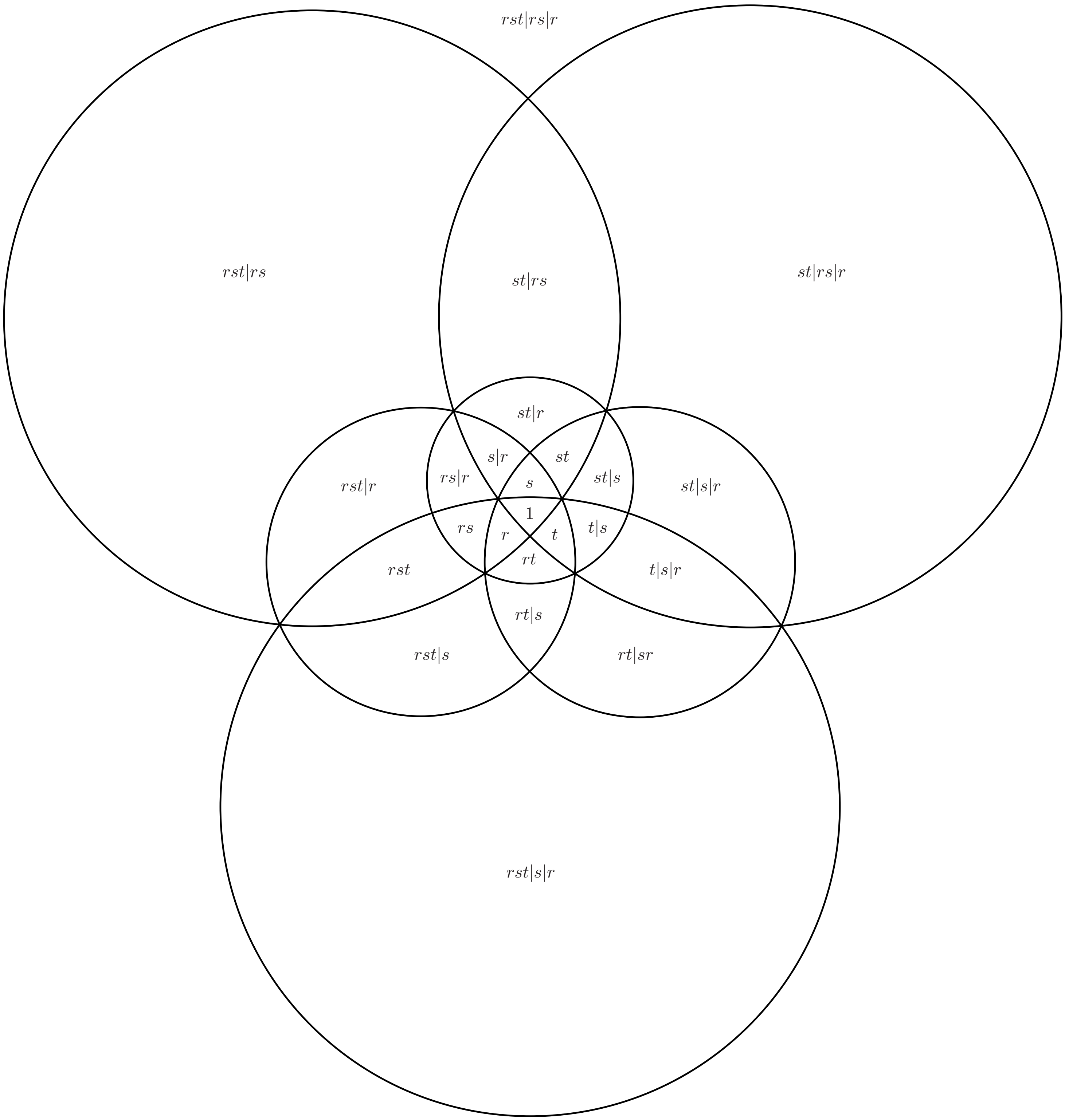

Figure 2. The fan defined by the reflecting hyperplanes for W of type A_3. The picture shows the intersection of this fan with the unit sphere, stereographically projected to the plane. Each region is labeled with the c-sorting word for the corresponding element (with dividers "|" retained). Here, as in Figure 1, $c = rst$.

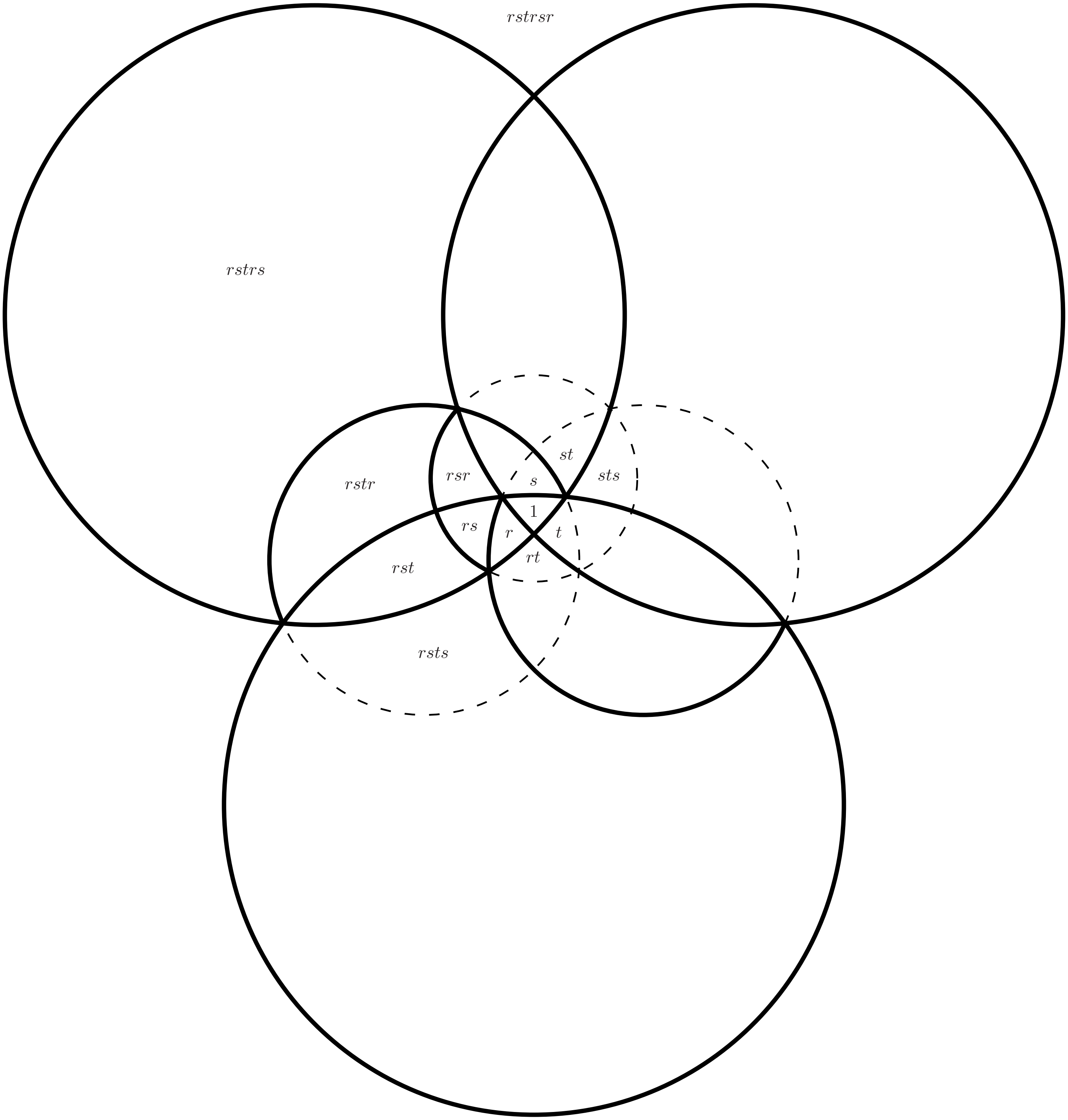

FIGURE 3. The c-Cambrian fan for W of type A_3 and $c = rst$. Solid lines indicate the maximal cones and dotted lines indicate the decomposition of each maximal cone into regions in the sense of Figure 2. The implied equivalence relation on W agrees with that shown in Figure 1. Each each maximal cone is labeled with the c-sorting word for the unique c-sortable element in the cone. (Dividers "|" are now dropped.)

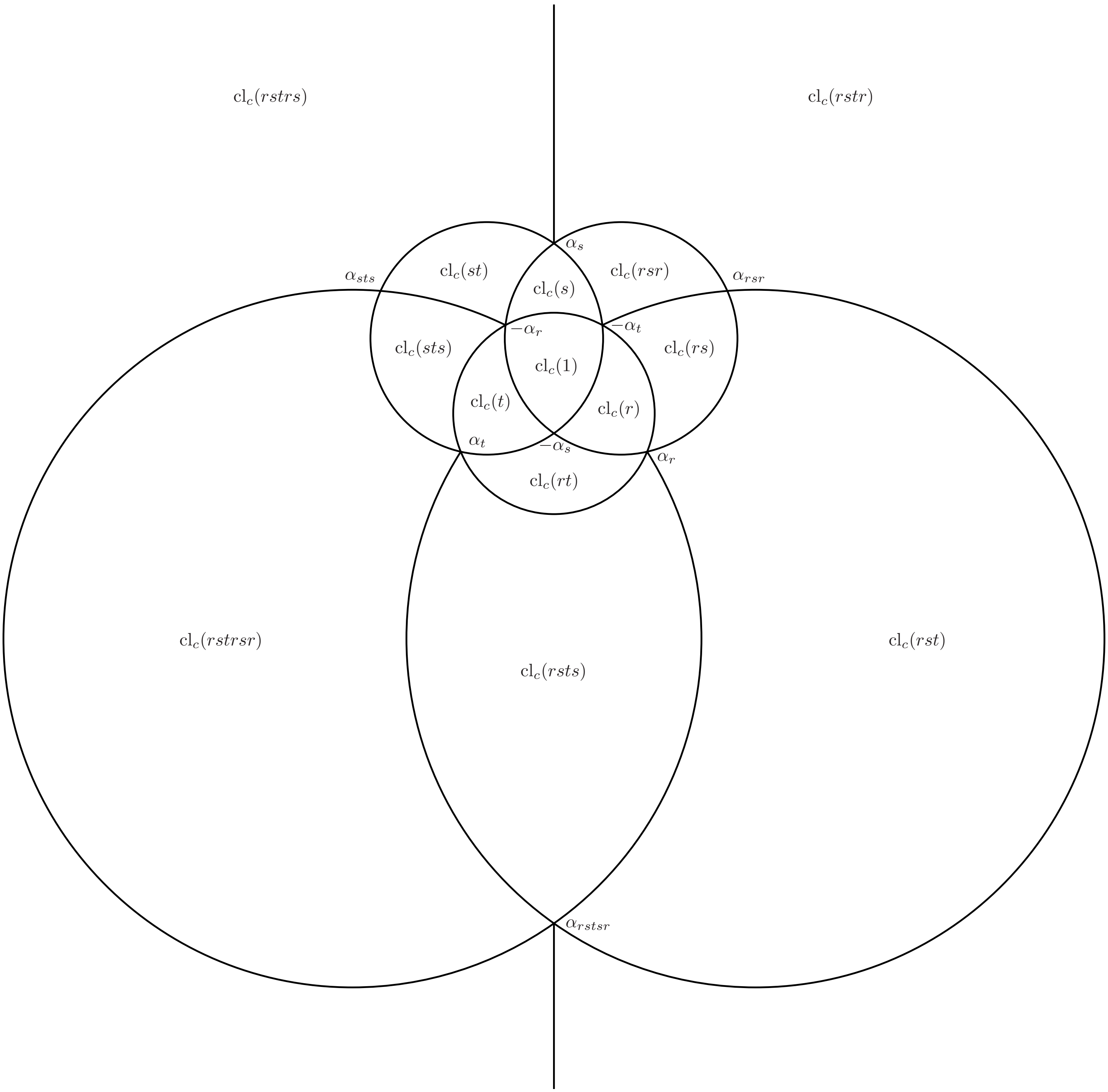

FIGURE 4. The c-cluster fan for W of type A_3 and $c = rst$. The almost positive roots are labeled. The roots α_s and α_{rstsr} are connected by an edge passing through the point at infinity. Each maximal cone of the fan is labeled $\mathrm{cl}_c(w)$ for the appropriate c-sortable element of W. (The adjacency graph of this fan is the exchange graph for cluster algebras of finite type A_3.)

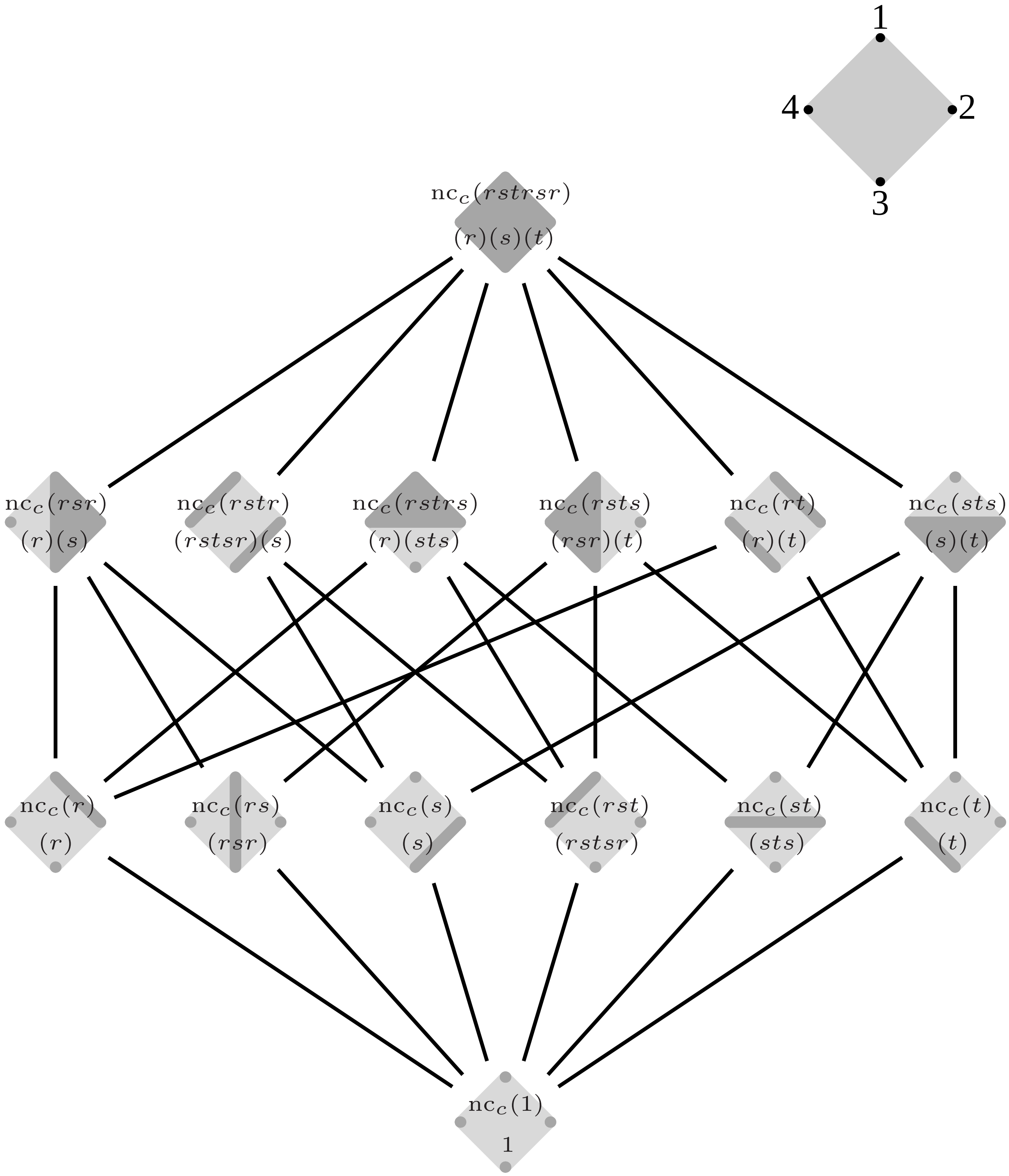

FIGURE 5. The c-noncrossing partition lattice for W of type A_3 and $c = rst$. Here $W = S_4$ with $r = (1\ 2)$, $s = (2\ 3)$, $t = (3\ 4)$ and $c = (1\ 2\ 3\ 4)$. Pictures at each vertex show a noncrossing diagram for each c-noncrossing partition, corresponding to an element x in $[1, c]_T$. Each picture is labeled by $\mathrm{nc}_c(w)$ for the appropriate c-sortable element w and by the c-noncrossing partition x. The latter is written using the word for x (in the alphabet T of reflections) arising from the map nc_c. Single reflections in this word are enclosed in parentheses; each reflection is represented as a word in the simple reflections $S = \{r, s, t\}$.