

Laplace transform “Bag of Tricks”

Most of these also appear in the Table of Laplace Transforms in the back of the book.

	Function-Land	Transform-World
	$f(t)$	$\mathcal{L}(f(t))$ or $F(s)$
Linearity of Transforms	$af(t) + bg(t)$	$aF(s) + bG(s)$
Transforms of Derivatives	$f'(t)$ $f''(t)$	$sF(s) - f(0)$ $s^2F(s) - sf(0) - f'(0)$
Translation on the s -Axis	$e^{at}f(t)$	$F(s - a)$
Transforms of Integrals	$\int_0^t f(\tau)d\tau$	$\frac{F(s)}{s}$
The Convolution Property	$f(t) * g(t)$	$F(s)G(s)$
Differentiation of Transforms	$-tf(t)$ $(-t)^n f(t)$	$F'(s)$ $F^{(n)}(s)$
Integration of Transforms	$\frac{f(t)}{t}$	$\int_s^\infty F(\sigma)d\sigma$
Translation on the t -Axis	$u(t - a)f(t - a)$	$e^{-as}F(s)$

Note that $f(t) * g(t)$ is defined to be $\int_0^t f(\tau)g(t - \tau)d\tau$ and this integral is what is shown in the table in the back of the book. For “Translation on the t -axis” to be helpful, you’ll need to know what $u(t - a)$ is.

These tricks all work for when $f(t)$ and $g(t)$ are piecewise continuous functions of exponential order as $t \rightarrow \infty$, **except:** 1. **Transforms of Derivatives:** $f(t)$ must be continuous and $f'(t)$ must be piecewise continuous and both must be of exponential order. 2. **Integration of Transforms:** $\frac{f(t)}{t}$ must also have a finite limit as $t \rightarrow 0$ from the right.