MATH 341 Fall 2023
Review: Material after Test 2

Below is a list of specific problems that one might reasonably expect to see on the second test. I don't promise that every exam problem will match with something on this sheet. Rather, my intention is for this to be an overview of the most important skills and understanding I expect you to have developed.

• Some important terms and concepts:

system of ODEs	matrix	vector
vector of functions (vector-valued function)	eigenvalue	eigenvector
derivative of a vector of functions	identity matrix	determinant

• Translate a linear system of first order ODEs into vector and matrix form, and translate in the other direction.
• Given an n^{th}-order ODE, write an equivalent system of n first-order ODEs.
• Solve a system of 2 (or 3) linear algebraic equations in 2 (or 3) unknowns.
• Carry out matrix addition and multiplication.
• Calculate the determinant of a 2×2 or 3×3 matrix.
• Given a matrix A, find the eigenvalues of A and find an eigenvector for each eigenvalue.
• Find the general solution to a homogeneous system of first order linear ODEs with constant coefficients by first solving the eigenvalue problem and then using the eigenvalues and eigenvectors to construct the solution. Then solve an initial value problem.
• Given an autonomous 2×2 system of ODEs (i.e. two equations for two unknown functions), find the critical points of the system.
• Given an autonomous 2×2 system of ODEs, sketch a phase portrait.
• Given an autonomous 2×2 system of ODEs, write down and solve the phase plane equation.
• Given a homogeneous 2×2 linear system with constant coefficients, classify the critical point at the origin as a node, saddle point, spiral point or center, and decide whether it is asymptotically stable, stable but not asymptotically stable or unstable.
• Draw a rough but qualitatively correct sketch of representative solutions of a given homogeneous 2×2 linear system with constant coefficients.

The following topics, although interesting, need not be a part of your test preparation: Matrix Exponentials (9.8); Almost linear systems (12.3), Series solutions of ODEs (8.3).