MATH 341, Fall 2023, QUIZ 9 ANSWERS

1. Use Laplace transform to solve the IVP $\quad y'' - y = 1, \quad y(0) = 0,\ y'(0) = 1$

ANSWER Transform the equation and then solve:

$$s^2Y(s) - 1 - Y(s) = \frac{1}{s}$$

$$(s^2 - 1)Y(s) = \frac{1}{s} + 1$$

$$(s^2 - 1)Y(s) = \frac{s+1}{s}$$

$$Y(s) = \frac{s+1}{s(s^2-1)}$$

$$Y(s) = \frac{s+1}{s(s+1)(s-1)}$$

$$Y(s) = \frac{1}{s(s-1)}$$

Now do partial fractions. You can use tricks, but there is also the "clear the denominators" method that I'll show here. (Multiply through by $s(s-1)$.)

$$\frac{1}{s(s-1)} = \frac{A}{s} + \frac{B}{s-1}$$

$$1 = A(s-1) + Bs$$

$$1 = (A+B)s - A$$

So $A + B = 0$ and $-A = 1$, so $A = -1$ and $B = 1$.

$$Y(s) = -\frac{1}{s} + \frac{1}{s-1}$$

$$y(t) = -1 + e^t$$

2. The following IVP describes a function $y(t)$. Transform the IVP to **write an equation for** the Laplace transform $Y(s)$. **Stop there!** You don't have to solve the equation for $Y(s)$.

$$y' + ty = e^t \cos t \qquad y(0) = 1$$

SCRATCH WORK:
$\mathcal{L}(y') = sY(s) - 1$ (Transforms of Derivatives)

$\mathcal{L}(ty) = -Y'(s)$ (Differentiation of Transforms)

$\mathcal{L}(\cos t) = \frac{s}{s^2+1}$ (Table)

$\mathcal{L}(e^t \cos t) = \frac{s-1}{(s-1)^2+1}$ (Translation on the s-Axis)

ANSWER: $\quad sY(s) - 1 - Y'(s) = \frac{s-1}{(s-1)^2+1}$.

You can stop there. But notice, this is a *differential equation* (like some of your homework problems).