MATH 341, Fall 2023, QUIZ 7 ANSWERS.

1. Use Linearity of Transforms and the partial table below to compute the following Laplace transforms. *No need to simplify.*

In these answers, letters like a, b, etc. are constants. Your quiz had actual numbers.

Laplace transform table

Function	Transform
$f(t)$	$F(s)$
1	$\frac{1}{s}$
e^{at}	$\frac{1}{s-a}$
t^n	$\frac{n!}{s^{n+1}}$
$\sin kt$	$\frac{k}{s^2+k^2}$
$\cos kt$	$\frac{s}{s^2+k^2}$

ANSWERS:

a. $\mathcal{L}\{a\} = \frac{a}{s}$.

b. $\mathcal{L}\left\{at^3 + \cos bt\right\} = \frac{6a}{s^4} + \frac{s}{s^2+b^2}$.

c. $\mathcal{L}\left\{e^{-at} - b\sin ct\right\} = \frac{1}{s+a} - \frac{bc}{s^2+c^2}$.

2. *Your quiz had an actual number for a.* Use the definition to find the Laplace transform of the function

$$f(t) = \begin{cases} 1 & \text{if } 0 \le t < 1 \\ a & \text{if } t > 1 \end{cases} .$$

For what s is $\mathcal{L}\left\{f(t)\right\}$ defined?

ANSWER:

$$\begin{aligned} \mathcal{L}\left\{f(t)\right\} &= \int_0^\infty e^{-st} f(t)\,\mathrm{d}t \\ &= \int_0^1 e^{-st}\,\mathrm{d}t + \int_1^\infty e^{-st} a\,\mathrm{d}t \\ &= \left[-\frac{1}{s}e^{-st}\right]_0^1 + \left[-\frac{a}{s}e^{-st}\right]_1^\infty \\ &= \left(\frac{-e^{-s}}{s} + \frac{1}{s}\right) + \left(0 + \frac{ae^{-s}}{s}\right) \\ &= \frac{(a-1)e^{-s}}{s} + \frac{1}{s} \end{aligned}$$

This is only defined (i.e. the second integral only converges) when $s > 0$.