MATH 341, Fall 2023, QUIZ 6 ANSWERS

In the following problems, you are given $y = c_1\cos(\omega t) + c_2\sin(\omega t)$ for specific numbers c_1, c_2, and ω, and you must rewrite it in the form $y = A\sin(\omega t+\phi)$ for specific numbers A, ω, and ϕ.
It will be helpful to know the trigonometric identity $\sin(a+b) = \sin b\cos a + \cos b\sin a$.

One version of the quiz:

$y = 3\sqrt{3}\cos(5t) - 3\sin(5t)$, or
$y = -3\sqrt{3}\cos(5t) + 3\sin(5t)$

ANSWER:
Using the trig identity, we see that $A\sin(\omega t+\phi) = A\sin\phi\cos(\omega t) + A\cos\phi\sin(\omega t)$. So, for one of the two problems we are looking for $A\sin\phi = 3\sqrt{3}$ and $A\cos\phi = -3$. Then $A = \sqrt{(A\sin\phi)^2 + (A\cos\phi)^2} = \sqrt{27+9} = 6$.

Also $\tan\phi = \frac{A\sin\phi}{A\cos\phi} = -\sqrt{3}$. Thinking about the unit circle (and/or right triangles), we see that ϕ is either $-\frac{\pi}{3}$ or $\frac{2\pi}{3}$. These two possibilities give the answers for these two problems: Since the angle $\frac{2\pi}{3}$ is in the 2nd quadrant, where cosine is negative and sine is positive, this would give $A\sin\phi = 3\sqrt{3}$ and $A\cos\phi = -3$, so

$$3\sqrt{3}\cos(5t) - 3\sin(5t) = 6\sin(5t + \frac{2\pi}{3}).$$

For the other problem, we want $A\sin\phi = -3\sqrt{3}$ and $A\cos\phi = 3$. The computation of A is the same, and still $\tan\phi = \frac{A\sin\phi}{A\cos\phi} = -\sqrt{3}$. The angle $-\frac{\pi}{3}$ is in the 4th quadrant, where cosine is positive and sine is negative, so this would give $A\sin\phi = -3\sqrt{3}$ and $A\cos\phi = 3$, and

$$-3\sqrt{3}\cos(5t) + 3\sin(5t) = 6\sin(5t - \frac{\pi}{3}).$$

If you didn't know that ϕ could be $-\frac{\pi}{3}$ or $\frac{2\pi}{3}$, you could use $\arctan(-\sqrt{3})$ in your answer for most of the credit. But you need to know that $\arctan(-\sqrt{3})$ is an angle in the 4th quadrant. So

$$3\sqrt{3}\cos(5t) - 3\sin(5t) = 6\sin(5t + \arctan(-\sqrt{3}) + \pi).$$

and

$$-3\sqrt{3}\cos(5t) + 3\sin(5t) = 6\sin(5t + \arctan(-\sqrt{3})).$$

The other version of the quiz:

$y = 3\cos(5t) - 3\sqrt{3}\sin(5t)$, or
$y = -3\cos(5t) + 3\sqrt{3}\sin(5t)$

ANSWER:
Using the trig identity, we see that $A\sin(\omega t+\phi) = A\sin\phi\cos(\omega t) + A\cos\phi\sin(\omega t)$. So, for one of the two problems we are looking for $A\sin\phi = 3$ and $A\cos\phi = -3\sqrt{3}$. Then $A = \sqrt{(A\sin\phi)^2 + (A\cos\phi)^2} = \sqrt{9+27} = 6$.

Also $\tan\phi = \frac{A\sin\phi}{A\cos\phi} = -\frac{1}{\sqrt{3}}$. Thinking about the unit circle (and/or right triangles), we see that ϕ is either $-\frac{\pi}{6}$ or $\frac{5\pi}{6}$. Since the angle $\frac{5\pi}{6}$ is in the 2nd quadrant, where cosine is negative and sine is positive, this would give $A \sin\phi = 3$ and $A\cos\phi = -3\sqrt{3}$, so

$$3\cos(5t) - 3\sqrt{3}\sin(5t) = 6\sin(5t + \frac{5\pi}{6}).$$

For the other problem, we want $A\sin\phi = -3$ and $A\cos\phi = 3\sqrt{3}$. The computation of A is the same, and still $\tan\phi = \frac{A\sin\phi}{A\cos\phi} = -\frac{1}{\sqrt{3}}$. The angle $-\frac{\pi}{6}$ is in the 4th quadrant, where cosine is positive and sine is negative, so this would give $A\sin\phi = -3$ and $A\cos\phi = 3\sqrt{3}$, and

$$-3\cos(5t) + 3\sqrt{3}\sin(5t) = 6\sin(5t - \frac{\pi}{6}).$$

noindent If you didn't know that ϕ could be $-\frac{\pi}{6}$ or $\frac{5\pi}{6}$, you could use $\arctan(-\frac{1}{\sqrt{3}})$ in your answer for most of the credit. But you need to know that $\arctan(-\frac{1}{\sqrt{3}})$ is an angle in the 4th quadrant. So

$$3\cos(5t) - 3\sqrt{3}\sin(5t) = 6\sin\Big(5t + \arctan\Big(-\frac{1}{\sqrt{3}}\Big) + \pi\Big).$$

and

$$-3\cos(5t) + 3\sqrt{3}\sin(5t) = 6\sin\Big(5t + \arctan\Big(-\frac{1}{\sqrt{3}}\Big)\Big).$$