MATH 341 Fall 2023, QUIZ 2 answers

1. Determine whether $y(x) = x^a$ is a solution to the ODE $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 + 2y^2 = b$.
(Your $a$ and $b$ varied.)

*We are* ***checking*** *a proposed solution, not* ***solving****, so we just see whether the proposed solution makes the equation true.* Compute (for your specific $a$):

$$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 + 2y^2 = a^2x^{2a-2} + 2x^{2a}$$

(For example, if you had $a = 3$, you got $9x^4 + 2x^6$.) That non-constant polynomial is not equal (as a function) to the constant number $b$.

*It's critical to be clear on what we mean by a solution: The equation says that two* ***functions*** *are equal, meaning that they are the same no matter what $x$ is. Yes, we can find, in principle, the values of $x$ that make the equation true, but that's not the point. The point is that as functions, $a^2x^{2a-2} + 2x^{2a}$ and $b$ are not the same.*

2. Determine whether the relationship $y^5 + y = x$ is an implicit solution to the ODE

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{5y^4 + 1}.$$

Differentiate both sides of $y^5 + y = x$ with respect to $x$. Don't forget the chain rule! You get:

$\frac{\mathrm{d}}{\mathrm{d}x}(y^5 + y) = \frac{\mathrm{d}}{\mathrm{d}x}x.$

$5y^4\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{\mathrm{d}y}{\mathrm{d}x} = 1.$

$\frac{\mathrm{d}y}{\mathrm{d}x}(5y^4 + 1) = 1.$

$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{5y^4+1}.$

Yes, this is an implicit solution to the ODE.

(Your quiz might have had different numbers, but the work was essentially the same and the answer was yes.)

3. $\lim_{x\to\infty} y(x) = 1$.
Or, in words, for large $x$, $y(x)$ approaches 1.

4. Fill in the table using Euler's method to approximate a solution to the initial value problem

$$y' = xy \qquad y(1) = 1$$

with step size $h = 1$. As in class and in the book, $y_n$ is the approximation to $y(x_n)$.

| $n$ | $x_n$ | $y_n$ | (Work) |
|---|---|---|---|
| 0 | 1 | 1 | |
| 1 | 2 | 2 | $1 + (1 \cdot 1) \cdot 1 = 2$ |
| 2 | 3 | 6 | $2 + (2 \cdot 2) \cdot 1 = 6$ |
| 3 | 4 | 24 | $6 + (3 \cdot 6) \cdot 1 = 24$ |

4. Fill in the table using Euler's method to approximate a solution to the initial value problem

$$y' = x + y \qquad y(1) = 2$$

with step size $h = 1$. As in class and in the book, $y_n$ is the approximation to $y(x_n)$.

| $n$ | $x_n$ | $y_n$ | (Work) |
|---|---|---|---|
| 0 | 1 | 2 | |
| 1 | 2 | 4 | $1 + (1 + 2) \cdot 1 = 4$ |
| 2 | 3 | 10 | $4 + (2 + 4) \cdot 1 = 10$ |
| 3 | 4 | 24 | $10 + (3 + 10) \cdot 1 = 23$ |