**Name:**

**Circle your recitation:**

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|---|---|---|---|
| 3A | 3C | 3B | 3D |
| Jennifer Buettner | | Matthew Elpers | |
| Erin Bednarek | | Julia Sanger | |
| 4A | 4C | 4B | 4D |

MATH 241, Spring 2023, TEST 2

PLEASE CHECK NOW that your exam has 4 pages (2 sheets of paper).
SHOW ALL OF YOUR WORK AND/OR EXPLAIN YOUR REASONING.

You **may or may not** need these facts on the test:

$$\int \frac{1}{x^2+a^2}\,\mathrm{d}x = \frac{1}{a}\arctan(\frac{x}{a}) + C$$

$$\sin^2 x = \frac{1}{2}(1-\cos(2x))$$

$$\cos^2 x = \frac{1}{2}(1+\cos(2x))$$

1. (15 points) Compute the indefinite integral

$$\int \tan^4 x \sec^4 x \,\mathrm{d}x$$

2. (15 points) Compute the indefinite integral

$$\int x^5 \left(\sqrt{1-x^2}\right)^3 \mathrm{d}x$$

3. Both parts of this problem are about the indefinite integral $\displaystyle\int \frac{x^3}{x^2+1}\,\mathrm{d}x$

a. (12 points) Make a trigonometric substitution to turn $\displaystyle\int \frac{x^3}{x^2+1}\,\mathrm{d}x$ into a trigonometric integral.
**Don't evaluate the resulting trigonometric integral. Just do the trig. substitution!**

b. (10 points) Evaluate the integral $\displaystyle\int \frac{x^3}{x^2+1}\,\mathrm{d}x$ by some method **other than trig. substitution.**

4. (10 points) Find the partial fractions decomposition of $\dfrac{x^2+9x+3}{(x^2+1)(x+4)}$.

**No integral here. Just do partial fractions!**

5. (10 points) Write the correct **form** for a partial fractions decomposition of $\dfrac{1}{x^3(x-1)^2(x^2+1)^2}$.

**Do not compute the partial fractions. Just write the correct form!**

6. (15 points) Consider a thin sheet of material cut out by $y = \sqrt{1-x^2}$ and $y = 0$ (a semicircle of radius 1.) Find the $y$-coordinate of the center of mass. (The $x$-coordinate is 0.) **You don't have to do an integral to know the total mass of the sheet!** The total mass will be $\rho$ times area (where $\rho$ is the area-density of the material), and the area of the semicircle is $\frac{\pi}{2}$, so the total mass is $\frac{\pi}{2}\rho$.

**Reminder:** The center of mass of a collection of masses is the weighted average of their positions, weighted by mass. For a sheet of material, we do the same thing, using integrals. For many purposes, we can treat the mass as if it is all located at the center of mass.

**Reminder of the error estimate for Simpson's Rule:**

Suppose we use Simpson's Rule to approximate $\displaystyle\int_a^b f(x)\,\mathrm{d}x$, breaking $[a,b]$ into $n$ intervals.

If $|f^{(4)}(x)| \le K$ for every $x \in [a,b]$, then the error is $\le \dfrac{K(b-a)^5}{180n^4}$.

7.

a. (7 points) Suppose for a certain $n$ that the error estimate above is 8. What does the error estimate become if we double $n$?

b. (6 points) Suppose I use Simpson's Rule with $n = 100$ to approximate $\displaystyle\int_0^2 3x^2\,\mathrm{d}x$. What answer does Simpson's Rule give? (Make a **simple** calculation and explain **briefly** why your calculation answers the question.)

CHALLENGE PROBLEM

Congratulations! You have finished the test. The following problem is a lot harder than the others, and is worth very few points of extra credit. Essentially correct answers will receive full credit and incorrect or incomplete answers will receive no credit. Don't waste time on this if you still have work or checking to do on other problems.

8. (3 points) Compute the indefinite integral $\int \tan^5 \theta \, \mathrm{d}\theta$ .

(I recommend using methods from two different topics that we covered, and using one of the methods backwards.)