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MATH 241, Spring 2023, TEST 1

PLEASE CHECK NOW that your exam has 4 pages (2 sheets of paper). For every problem on the test, SHOW ALL OF YOUR WORK AND/OR EXPLAIN YOUR REASONING.

1. (10 points) Draw a rough sketch of a continuous function f(x) that fits the following descriptions. (One sketch of one function that satisfies all of the descriptions.)

- f(3) = -2.
- f'(3) = 0 but f'(x) < 0 whenever $x \neq 3$.
- f''(x) > 0 for all x < 3 and f''(x) < 0 for x > 3.



2. (5 points each) Compute the following derivatives.

a.
$$\frac{\mathrm{d}}{\mathrm{d}x} \frac{x}{x^2 + 7}$$

b.
$$\frac{\mathrm{d}}{\mathrm{d}x} e^{3x^2-4}$$

c.
$$\frac{\mathrm{d}}{\mathrm{d}x}$$
 $(\ln x) \cdot (\cos(6x))$

3. (5 points each) Compute the following (definite and indefinite) integrals.

a.
$$\int x \cos(2x) \, \mathrm{d}x$$

b.
$$\int \sin x \cos^2(x) \, \mathrm{d}x$$

c.
$$\int_0^{\frac{\pi}{5}} \sin(5x) \,\mathrm{d}x$$

4. (15 points) Set up but do not evaluate an integral that computes the arc length of graph of the function $y = \cos x$ from x = 0 to x = 3.

5. (15 points) Use an integral to find the average value of the function $f(x) = e^x - e^{-x}$ on the interval [-5, 5]. (You might see a way of doing this without an integral. If so, use that way to decide if your answer makes sense. You still **must** show how to compute this with an integral. Yes, you can do the integral without a calculator.) 6. (15 points) A mass moves along a line, and we use x for its position on the line in meters. There is a force (in Newtons) acting on the mass that depends only on x, and given by $F(x) = x^3$ (pointing in the direction of positive x). Find the work done by the force as the mass moves from x = 0 to x = 2. Explain your formula briefly.

Reminder: The work done by a *constant* force through some displacement is Force \times displacement. Units are N-m (Newton-meters) or equivalently, J (Joules).

7. (15 points) A cylindrical tank of radius 2 is lying with its axis horizontal, so that each circular side is in a vertical plane. The tank is exactly half full of water. Find the force on one of the circular sides due to water pressure. Leave your answer as some number times 9800.

Reminder: Water pressure in N/m^2 is 9800 times depth (when depth is measured in m). Pressure is Force per unit area.

CHALLENGE PROBLEMS Congratulations! You have finished the test. Don't waste time on the challenge problems if you still have work or checking to do on other problems.

8. (3 points) Recall that polar coordinates (r, θ) are related to Cartesian coordinates by $x = r \cos \theta$ and $y = r \sin \theta$. Give a formula for the arc length of a curve in the plane described in polar coordinates by the equation $r = f(\theta)$ for θ from a to b. For full credit, simplify.

9. (3 points) Given a continuous function f(x), define a function a where a(x) is the average of f on the interval [0, x]. Write an expression for $\frac{\mathrm{d}a}{\mathrm{d}x}$. For full credit, simplify.