MATH 241, Spring 2023, Nathan Reading
Review: Test 3

This is an overview of the most important skills and understanding I expect you to have developed. I don't promise that every exam problem will match with something on this sheet.

Exam questions might test your understanding of some of the following important terms and concepts:

differential equation	ODE	order of an ODE
solution of an ODE	independent variable	general solution of an ODE
initial condition	initial value problem	IVP
implicit differentiation	implicit solution to an ODE	slope field
2^{nd}-order linear ODE	homogeneous	non-homogeneous
complex number		

The exam might test your ability to carry out some of the following procedures.

- Given an ODE or IVP and a function, text whether the function is a solution to the ODE/IVP.
- Given an ODE of the form $y' = f(x, y)$, draw a slope field for the ODE (or demonstrate your ability to do so).
- Given the slope field of an ODE, sketch a plausible solution to a given IVP.
- Carry out several steps of Euler's Method.
- Solve a separable ODE. You will not necessarily be told that the ODE is separable. Often, the solution is an implicit solution. You will be given guidance about whether you need to write an explicit solution.
- Given a family of curves, find the orthogonal trajectories.
- Solve a tank problem (AKA mixing problem). You will do this by writing an IVP (ODE + initial condition) to describe the situation. Then you will solve the IVP. You might also need to use the solution to answer questions about the tank problem. (You might be asked to do only part of this, like write down the IVP but don't solve.)
- Recognize a 2^{nd}-order linear ODE and tell whether it is homogeneous or non-homogeneous.
- Find the general solution to a 2^{nd}-order homogeneous linear ODE with constant coefficients, using the Auxiliary Equation Method.
- Solve an IVP for a 2^{nd}-order homogeneous linear ODE with constant coefficients.
- Find the general solution to a 2^{nd}-order non-homogeneous linear ODE with constant coefficients by first solving the complementary ODE (the associated homogeneous ODE), then finding any particular solution to the non-homogenous ODE using the Method of Undetermined Coefficients, and finally, combining these solutions correctly.
- Solve an IVP for a 2^{nd}-order non-homogeneous linear ODE with constant coefficients.

Some important ideas and examples:

- As a rule of thumb, for a k^{th}-order ODE, we expect the general solution to have k arbitrary constants. (Things can go wrong, though, but we won't deal with any situations where they go wrong.)
- Exponential growth/decay.*
- The logistic model.**
- Newton's Law of Cooling.**
- Mass-on-a spring.***
- Circuits with resistors, capacitors and/or inductors in series.***

*I will not ask you to do the type of pre-calc exponential growth problems that were in homework.
**I will not ask you to fit to a logistic model, and we won't worry about Newton's Law of Cooling.
***I will not ask you to deal directly with springs or circuits.

Challenge Problems:

I will ask a few challenge questions at the end of the exam, worth very few points, and these problems are exempt from anything I have told you or will tell you about what I will or will not ask on the test.

Closed book, closed notes, no calculators.