MA 241, Spring 2023, QUIZ 9 answers.

General Comment: This quiz was designed to find *basic* misunderstandings that you might have. (Test problems might be more involved than these problems.) I think the quiz found at least one basic misunderstanding that many of you have: You are unclear about the difference between a *series*, the *sequence* of its terms, and the *sequence of partial sums*. This revealed itself if you wrote the 3rd *term* in Problem 1 rather than the 3rd partial sum. It also revealed itself if you wrote that $\sum_{n=1}^{\infty} \frac{6}{n^7}$ converges to 0. Yes, it converges, but it's a sum of positive numbers, and they can't add up to 0. (The sequence of terms $\frac{6}{n^7}$ *does* converge to 0, but that's not the same thing.)

1. Find the 3rd partial sum of the series $\sum_{n=1}^{\infty}(n+1)$ $\quad S_3 = 2+3+4 = 9$.

1. Find the 3rd partial sum of the series $\sum_{n=1}^{\infty}(n+2)$ $\quad S_3 = 3+4+5 = 12$.

1. Find the 3rd partial sum of the series $\sum_{n=1}^{\infty}(n+3)$ $\quad S_3 = 4+5+6 = 15$.

1. Find the 3rd partial sum of the series $\sum_{n=1}^{\infty}(n+4)$ $\quad S_3 = 5+6+7 = 18$.

2. For each series, decide whether it converges or diverges and if it converges, give the sum, **if you can**.

$\sum_{n=1}^{\infty} \left(\frac{6}{5}\right)^n$ DIVERGES (Geometric Series, $|r| > 1$)

$\sum_{n=1}^{\infty} \frac{n}{2}$ DIVERGES (Test for Divergence, terms do not limit to 0).

$\sum_{n=1}^{\infty} \frac{2}{\sqrt[3]{n}}$ DIVERGES (p-Series, $p < 1$).

$\sum_{n=1}^{\infty} \frac{6}{n^7}$ CONVERGES (p-Series, $p > 1$).

$\sum_{n=1}^{\infty} \frac{1}{n}$ DIVERGES (Harmonic Series, AKA p-Series, $p = 1$)

$\sum_{n=1}^{\infty} \left(\frac{1}{5}\right)^n$ CONVERGES to $\frac{\frac{1}{5}}{1-\frac{1}{5}} = \frac{1}{4}$ (Geometric Series, $a = \frac{1}{5}$, $r = \frac{1}{5}$)

$\sum_{n=1}^{\infty} \left(\frac{2}{5}\right)^n$ CONVERGES to $\frac{\frac{2}{5}}{1-\frac{2}{5}} = \frac{2}{3}$ (Geometric Series, $a = \frac{2}{5}$, $r = \frac{2}{5}$)

$\sum_{n=1}^{\infty} \left(\frac{3}{5}\right)^n$ CONVERGES to $\frac{\frac{3}{5}}{1-\frac{3}{5}} = \frac{3}{2}$ (Geometric Series, $a = \frac{3}{5}$, $r = \frac{3}{5}$)

$\sum_{n=1}^{\infty} \left(\frac{4}{5}\right)^n$ CONVERGES to $\frac{\frac{4}{5}}{1-\frac{4}{5}} = 4$ (Geometric Series, $a = \frac{4}{5}$, $r = \frac{4}{5}$)

3. Which of these describes a correct use of the Comparison Test? *In both situations, all of the terms a_n and b_n are positive.* Over all the quizzes, there were 4 situations. I will show all 4 and write CORRECT or INCORRECT.

Situation 1: I know that $\sum_{n=1}^{\infty} b_n$ converges and I know that $a_n \le b_n$ for all $n \ge 1$. Therefore $\sum_{n=1}^{\infty} a_n$ converges. CORRECT

Situation 2: I know that $\sum_{n=1}^{\infty} b_n$ converges and I know that $a_n \geq b_n$ for all $n \geq 1$. Therefore $\sum_{n=1}^{\infty} a_n$ converges. INCORRECT

Situation 3: I know that $\sum_{n=1}^{\infty} b_n$ diverges and I know that $a_n \leq b_n$ for all $n \geq 1$. Therefore $\sum_{n=1}^{\infty} a_n$ diverges. INCORRECT

Situation 4: I know that $\sum_{n=1}^{\infty} b_n$ diverges and I know that $a_n \geq b_n$ for all $n \geq 1$. Therefore $\sum_{n=1}^{\infty} a_n$ diverges. CORRECT