MATH 241, Fall 2018, QUIZ 9 answers

1. Find the general solution for each of the following ODEs.

$y'' - 2y' + 5y = 0$
Auxiliary equation: $r^2 - 2r + 5 = 0$. Solve (quadratic formula): $r = 1 \pm 2i$.
General solution: $y(x) = C_1 e^x \cos 2x + C_2 e^x \sin 2x$.

$y'' - 2y' + y = 0$
Auxiliary equation: $r^2 - 2r + 1 = 0$. Solve (factor or quadratic formula): $r = 1$.
General solution: $y(x) = C_1 e^x + C_2 x e^x$.

$y'' - 2y' - 3y = 0$
Auxiliary equation: $r^2 - 2r - 3 = 0$. Solve (factor or quadratic formula): $r = -1, 3$.
General solution: $y(x) = C_1 e^{-x} + C_2 e^{3x}$.

2. Suppose a certain 2nd-order linear ODE has general solution $y = c_1 e^{2x} + c_2 e^{5x}$.
Solve the initial value problem for that ODE and initial conditions $y(0) = 0$ and $y'(0) = 3$.
$y(0) = c_1 + c_2 = 0$, so $c_1 = -c_2$.

$y' = 2c_1 e^{2x} + 5c_2 e^{5x}$
$y'(0) = 2c_1 + 5c_2 = 3$
Replacing c_1 by $-c_2$, we get $-2c_2 + 5c_2 = 3$, so $3c_2 = 3$, so $c_2 = 1$ and $c_1 = -1$.
$\boxed{y = -e^{2x} + e^{5x}}$

2. Suppose a certain 2nd-order linear ODE has general solution $y = c_1 e^{3x} + c_2 e^{4x}$.
Solve the initial value problem for that ODE and initial conditions $y(0) = 0$ and $y'(0) = 2$.
$y(0) = c_1 + c_2 = 0$, so $c_1 = -c_2$.

$y' = 3c_1 e^{3x} + 4c_2 e^{4x}$
$y'(0) = 3c_1 + 4c_2 = 2$
Replacing c_1 by $-c_2$, we get $-3c_2 + 4c_2 = 2$, so $c_2 = 2$ and $c_1 = -2$.
$\boxed{y = -2e^{3x} + 2e^{4x}}$

2. Suppose a certain 2nd-order linear ODE has general solution $y = c_1 e^{6x} + c_2 e^{x}$.
Solve the initial value problem for that ODE and initial conditions $y(0) = 0$ and $y'(0) = 5$.
$y(0) = c_1 + c_2 = 0$, so $c_1 = -c_2$.

$y' = 6c_1 e^{6x} + c_2 e^{x}$
$y'(0) = 6c_1 + c_2 = 5$
Replacing c_1 by $-c_2$, we get $-6c_2 + c_2 = 5$, so $-5c_2 = 5$, so $c_2 = -1$ and $c_1 = 1$.
$\boxed{y = e^{6x} - e^{x}}$