MATH 241, Spring 2023, QUIZ 6 answers

1. The perpendicular line has slope that is the *negative reciprocal* of the slope of the given line. The answers to the various versions of the problem were:

Given slope	perpendicular slope
3	$-\frac{1}{3}$
$-\frac{1}{4}$	4
-3	$\frac{1}{3}$
$\frac{1}{4}$	-4

2. There were 4 versions of the problem.

$y' = \dfrac{x^2}{y^7}$	$y' = \dfrac{x^4}{y^5}$	$y' = \dfrac{x^3}{y^6}$	$y' = \dfrac{x^5}{y^4}$	Separate
$y^7\,\mathrm{d}y = x^2\,\mathrm{d}x$	$y^5\,\mathrm{d}y = x^4\,\mathrm{d}x$	$y^6\,\mathrm{d}y = x^3\,\mathrm{d}x$	$y^4\,\mathrm{d}y = x^5\,\mathrm{d}x$	
$\int y^7\,\mathrm{d}y = \int x^2\,\mathrm{d}x$	$\int y^5\,\mathrm{d}y = \int x^4\,\mathrm{d}x$	$\int y^6\,\mathrm{d}y = \int x^3\,\mathrm{d}x$	$\int y^4\,\mathrm{d}y = \int x^5\,\mathrm{d}x$	Integrate
$\dfrac{y^8}{8} = \dfrac{x^3}{3} + C$	$\dfrac{y^6}{6} = \dfrac{x^5}{5} + C$	$\dfrac{y^7}{7} = \dfrac{x^4}{4} + C$	$\dfrac{y^5}{5} = \dfrac{x^6}{6} + C$	The answer
$3y^8 = 8x^3 + C$	$5y^6 = 6x^5 + C$	$4y^7 = 7x^4 + C$	$6y^5 = 5x^6 + C$	

The last step, simplifying by clearing denonimators, is optional. The "C" in the last step is different from the "C" in the next-to-last step, as we discussed several times in class.

3. A tank initially contains 200 gallons of saltwater with a total of 50 kg of salt. Saltwater with a concentration of 2 kg per gallon flows in at 3 gallons per minute and the perfectly mixed solution flows out at the same rate.

Let $S(t)$ be the amount (in kg) of salt in the tank at time t (in minutes).
The situation is described by the following initial value problem:

$$\boxed{\frac{\mathrm{d}S}{\mathrm{d}t} = 6 - \frac{3S}{200} \qquad S(0) = 50.}$$

Explanation:

The ODE is

$$\frac{\mathrm{d}S}{\mathrm{d}t} = (\text{rate in}) - (\text{rate out}) = 2\frac{\text{kg}}{\text{gallon}} \cdot 3\frac{\text{gallons}}{\text{minute}} - \frac{S}{200}\frac{\text{kg}}{\text{gallon}} \cdot 3\frac{\text{gallons}}{\text{minute}}$$

The initial condition is just part of the given information: the tank initially contains 50 kg. If you had $S(0) = \frac{50}{200}$, you would have had serious problems if I had asked you to solve the IVP. The ODE you wrote (if you wrote what I wrote) described the **amount** of salt (in kg), not the concentration (in kg/gallon). So $S(0)$ is an amount, not a concentration.

That brings up a really important point: Make sure you define what your quantities are. You should even **write down** what your quantities are (in this class and in classes and work situation where you use this math!). If S is concentration, we have to write a different ODE (and we never talked about that). If you thought S was a concentration and you wrote the correct ODE for amounts, then you don't know why you wrote down that ODE.

Other versions of the problem:

3. A tank initially contains 300 gallons of saltwater with a total of 40 kg of salt. Saltwater with a concentration of 2 kg per gallon flows in at 5 gallons per minute and the perfectly mixed solution flows out at the same rate.

$\frac{\mathrm{d}S}{\mathrm{d}t} = (\text{rate in}) - (\text{rate out}) = 2\frac{\text{kg}}{\text{gallon}} \cdot 5\frac{\text{gallons}}{\text{minute}} - \frac{S}{300}\frac{\text{kg}}{\text{gallon}} \cdot 5\frac{\text{gallons}}{\text{minute}}$

$\boxed{\frac{\mathrm{d}S}{\mathrm{d}t} = 10 - \frac{S}{60} \qquad S(0) = 40.}$

3. A tank initially contains 400 gallons of saltwater with a total of 30 kg of salt. Saltwater with a concentration of 2 kg per gallon flows in at 2 gallons per minute and the perfectly mixed solution flows out at the same rate.

$\frac{\mathrm{d}S}{\mathrm{d}t} = (\text{rate in}) - (\text{rate out}) = 2\frac{\text{kg}}{\text{gallon}} \cdot 2\frac{\text{gallons}}{\text{minute}} - \frac{S}{400}\frac{\text{kg}}{\text{gallon}} \cdot 2\frac{\text{gallons}}{\text{minute}}$

$\boxed{\frac{\mathrm{d}S}{\mathrm{d}t} = 4 - \frac{S}{200} \qquad S(0) = 30.}$

3. A tank initially contains 500 gallons of saltwater with a total of 20 kg of salt. Saltwater with a concentration of 2 kg per gallon flows in at 4 gallons per minute and the perfectly mixed solution flows out at the same rate.

$\frac{\mathrm{d}S}{\mathrm{d}t} = (\text{rate in}) - (\text{rate out}) = 2\frac{\text{kg}}{\text{gallon}} \cdot 4\frac{\text{gallons}}{\text{minute}} - \frac{S}{500}\frac{\text{kg}}{\text{gallon}} \cdot 2\frac{\text{gallons}}{\text{minute}}$

$\boxed{\frac{\mathrm{d}S}{\mathrm{d}t} = 8 - \frac{S}{250} \qquad S(0) = 20.}$