MATH 241, Spring 2023, QUIZ 2 answers.
Yes, I did take off points if you did not write your name or circle your recitation.

NOTE: Your problems may have been slightly different (i.e. slightly different numbers), but the differences should be small enough that these solutions will tell you how to do the problem. You are always welcome to ask me (or a TA) for details about your specific problem.

1. Find the arc length of the parametrized curve given by $x(t) = e^{3t}$ and $y(t) = \sin t$ from $t = 1$ to $t = 5$. The general formula is $\int_a^b \sqrt{(\frac{\mathrm{d}x}{\mathrm{d}t})^2 + (\frac{\mathrm{d}y}{\mathrm{d}t})^2}\ \mathrm{d}t$. So for this problem, the arc length is

$$\int_0^2 \sqrt{(3e^{3t})^2 + (\cos t)^2}\ \mathrm{d}t = \int_1^5 \sqrt{9e^{6t} + \cos^2 t}\ \mathrm{d}t.$$

You should probably do these kinds of basic simplifications, but I didn't take off points this time. I **did** take off a point for leaving off "dt".

If you had trouble remembering the formula for arc length (like if you thought there should be a $\frac{1}{b-a}$ in front of the integral, my advice is to think through (and/or read through) again where the formula comes from.

2. The average value of the function $f(x) = 3x^2$ on the interval $[-3, 1]$ is

$$\frac{1}{1-(-3)} \int_{-3}^{1} 3x^2\,\mathrm{d}x = \frac{1}{4}\left[x^3\right]_{-3}^{1} = \frac{1}{4}(1-(-27)) = \frac{28}{4} = 7.$$

If you got a negative answer, you had a chance to notice that it was wrong: Since $3x^2$ is never negative, its average can't be negative.

I **did** take off a point for leaving $\frac{9}{3}$ or $\frac{28}{4}$.