1.
$$\int \frac{4x-6}{x^2-9} dx$$
 Factor denominator: $x^2-9 = (x+3)(x-3)$
Partial fractions: $\frac{4x-6}{x^2-9} = \frac{A}{x+3} + \frac{B}{x-3}$

Use tricks if you like, but one way to find A and B is to clear denominators:

$$4x + 6 = A(x - 3) + B(x + 3) = Ax - 3A + Bx + 3B = (A + B)x + (3B - 3A),$$

so A + B = 4 and 3B - 3A = -6.

We solve to get A = 3 and B = 1.

$$\int \frac{4x-6}{x^2-9} \,\mathrm{d}x = \int \frac{3}{x+3} \,\mathrm{d}x + \int \frac{1}{x-3} \,\mathrm{d}x = 3\ln|x+3| + \ln|x-3| + C.$$

COMMENTS ON PROBLEM 1:

• The partial fractions were not $\frac{Ax+B}{x^2-9}$ because x^2-9 is not an **indecomposable** quadratic factor.

It factors as (x+3)(x-3). In fact, if the partial fractions were $\frac{Ax+B}{x^2-9}$, you would already know A = 4 and B = -6. But you need to find a partial fractions decomposition $\frac{A}{x+3} + \frac{B}{x-3}$ because otherwise you can't integrate!

• No polynomial long division. Degree of the numerator is already less than degree of the denominator.



 $= \arctan x + C$