MATH 141, Fall 2025, QUIZ 7 solutions

1. If $A^2 - 3 = B^5$ and $\frac{\mathrm{d}A}{\mathrm{d}t} = 1$, find $\frac{\mathrm{d}B}{\mathrm{d}t}$ when $A = 2$.

Differentiate both sides (using chain rule!) to get a relationship between the rates: $2A\frac{\mathrm{d}A}{\mathrm{d}t} = 5B^4\frac{\mathrm{d}B}{\mathrm{d}t}$.

When $A = 2$, $2^2 - 3 = 1 = B^5$, so $B = 1$.

Put all this into the relationship between the rates, then solve: $2 \cdot 2 \cdot 1 = 5 \cdot 1^4 \frac{\mathrm{d}B}{\mathrm{d}t}$ $\boxed{\frac{\mathrm{d}B}{\mathrm{d}t} = \frac{4}{5}}$.

1. If $B^2 - 3 = A^3$ and $\frac{\mathrm{d}A}{\mathrm{d}t} = 1$, find $\frac{\mathrm{d}B}{\mathrm{d}t}$ when $B = 2$.

Differentiate both sides (using chain rule!) to get a relationship between the rates: $2B\frac{\mathrm{d}B}{\mathrm{d}t} = 3A^2\frac{\mathrm{d}A}{\mathrm{d}t}$.

When $B = 2$, $2^2 - 3 = 1 = B^3$, so $A = 1$.

Put all this into the relationship between the rates, then solve: $2 \cdot 2 \cdot \frac{\mathrm{d}B}{\mathrm{d}t} = 3 \cdot 1^2 \cdot 1$ $\boxed{\frac{\mathrm{d}B}{\mathrm{d}t} = \frac{3}{4}}$.

1. If $A^2 - 8 = B^3$ and $\frac{\mathrm{d}A}{\mathrm{d}t} = 1$, find $\frac{\mathrm{d}B}{\mathrm{d}t}$ when $A = 3$.

Differentiate both sides (using chain rule!) to get a relationship between the rates: $2A\frac{\mathrm{d}A}{\mathrm{d}t} = 3B^2\frac{\mathrm{d}B}{\mathrm{d}t}$.

When $A = 3$, $3^2 - 8 = 1 = B^3$, so $B = 1$.

Put all this into the relationship between the rates, then solve: $2 \cdot 3 \cdot 1 = 3 \cdot 1^2 \frac{\mathrm{d}B}{\mathrm{d}t}$ $\boxed{\frac{\mathrm{d}B}{\mathrm{d}t} = 2}$.

1. If $B^2 - 8 = A^5$ and $\frac{\mathrm{d}A}{\mathrm{d}t} = 1$, find $\frac{\mathrm{d}B}{\mathrm{d}t}$ when $B = 3$.

Differentiate both sides (using chain rule!) to get a relationship between the rates: $2B\frac{\mathrm{d}B}{\mathrm{d}t} = 5A^4\frac{\mathrm{d}A}{\mathrm{d}t}$.

When $B = 3$, $3^2 - 8 = 1 = A^5$, so $A = 1$.

Put all this into the relationship between the rates, then solve: $2 \cdot 3 \cdot \frac{\mathrm{d}B}{\mathrm{d}t} = 5 \cdot 1^4 \cdot 1$ $\boxed{\frac{\mathrm{d}B}{\mathrm{d}t} = \frac{5}{6}}$.

2. Let $f(x) = x^4 + x^3 + x^2 + 2x + 4$. Find the linear approximation $L_1(x)$ to $f(x)$ at $x = 1$.

$f'(x) = 4x^3 + 3x^2 + 2x + 2$, so $f'(1) = 4 + 3 + 2 + 2 = 11$. Also $f(1) = 9$. $\boxed{L_1(x) = 9 + 11(x-1)}$

This is $L_c(x) = f(c) + f'(c)(x - c)$ *for this* f *and for* $x = 1$.

Alternate versions of the problem and answers (work is similar):

$f(x) = x^4 + x^3 + 2x^2 + x + 3 \quad \to \quad L_1(x) = 8 + 12(x - 1)$.

$f(x) = x^4 + 2x^3 + x^2 + x + 2 \quad \to \quad L_1(x) = 7 + 13(x - 1)$.

$f(x) = 2x^4 + x^3 + x^2 + x + 1 \quad \to \quad L_1(x) = 6 + 14(x - 1)$.

3. (5 points) Find the linear approximation to $\sqrt[3]{x}$ at $x = 8$ and use it to approximate $\sqrt[3]{8.12}$.

(*You will make your calculator-free computations much easier if you think of* 8.12 *as* $8 + \frac{12}{100}$.)

$f(x) = \sqrt[3]{x}$, so $f(8) = 2$. $f'(x) = \frac{1}{3}x^{-\frac{2}{3}} = \frac{1}{3(\sqrt[3]{x})^2}$. $f'(8) = \frac{1}{3(\sqrt[3]{8})^2} = \frac{1}{12}$

$L_8(x) = 2 + \frac{1}{12}(x - 8)$. *This is* $L_c(x) = f(c) + f'(c)(x - c)$ *for this* f *and for* $x = 8$.

The approximation is $\sqrt[3]{x} \approx 2 + \frac{1}{12}(x - 8)$. *You could simplify this, but because you're taking* x*-values close to* x*, simplifying, in this case, gives you something less easy to use!*

$$\boxed{\sqrt[3]{8.12} \approx 2 + \frac{1}{12}\left(\frac{12}{100}\right) = 2 + \frac{1}{100} = 2.01}$$