MATH 141, Fall 2025, QUIZ 6 ANSWERS. *In a fit of temporary lunacy (and worrying about whether the quiz was too long for 12 minutes... it wasn't)* I made the quiz worth 11 points, so that if you only started Problem 3, you could still get a "10". So I left it that way. It's out of 11, but the grades will treat it like it's out of 10 (i.e. an 11 is like getting 110% on a test).

1.a. $\frac{\mathrm{d}}{\mathrm{d}x}\, e^x = e^x$

1.b. $\frac{\mathrm{d}}{\mathrm{d}x}\, \ln x = \dfrac{1}{x}$

1.c. $\frac{\mathrm{d}}{\mathrm{d}x}\, \dfrac{\ln(ax)}{x} = \dfrac{x \cdot \frac{1}{x} - \ln(ax) \cdot 1}{x^2} = \dfrac{1 - \ln(ax)}{x^2}$ *You had a specific number for a.*

As part of the quotient rule, you had to find the derivative of $\ln(ax)$. That's easy, but surprising, by the chain rule: $\dfrac{\mathrm{d}}{\mathrm{d}x} \ln(ax) = \frac{1}{ax} \cdot a = \frac{1}{x}$. Another way to see that is by logarithm rules: $\ln(ax)$ is $\ln a + \ln x$. Since $\ln(ax)$ differs from $\ln x$ by adding a constant, they have the same derivative.

1.d. $\frac{\mathrm{d}}{\mathrm{d}x}\, e^{ax} = e^{ax} \cdot a$, more nicely written as ae^{ax}. *You had a specific number for a.*

1.e. $\frac{\mathrm{d}}{\mathrm{d}x}\, (e^x \ln x) = e^x \cdot \dfrac{1}{x} + e^x \ln x = \dfrac{e^x}{x} + e^x \ln x$

2. $xy^2 = ay^3 + x$. *You had a specific number for a.*

a. Differentiate both sides $\dfrac{\mathrm{d}}{\mathrm{d}x}$: $1 \cdot y^2 + x \cdot 2y \dfrac{\mathrm{d}y}{\mathrm{d}x} = 3ay^2 \dfrac{\mathrm{d}y}{\mathrm{d}x} + 1.$

Simplify: $y^2 + 2xy \dfrac{\mathrm{d}y}{\mathrm{d}x} = 3ay^2 \dfrac{\mathrm{d}y}{\mathrm{d}x} + 1$

Solve for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$: $2xy \dfrac{\mathrm{d}y}{\mathrm{d}x} - 3ay^2 \dfrac{\mathrm{d}y}{\mathrm{d}x} = 1 - y^2$ $\qquad \dfrac{\mathrm{d}y}{\mathrm{d}x}(2xy - 3ay^2) = 1 - y^2$ $\qquad \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1 - y^2}{2xy - 3ay^2}$

The extra question that wasn't part of the quiz: "Find the point on the curve with $y = 2$ and the slope of the tangent line to the curve at that point." This question would work best with $a = 3$.

Using the original equation: $x \cdot 2^2 = 3 \cdot 2^3 + x$ $\qquad 4x = 24 + x$ $\qquad x = 8$

At the point $(8, 2)$: $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1 - 2^2}{2 \cdot 8 \cdot 2 - 9 \cdot 2^2} = \dfrac{-3}{-4} = \dfrac{3}{4}.$ *That's the slope of the tangent line.*

Point-slope formula for a line through $(8, 2)$ with slope $\frac{3}{4}$: $\qquad y - 2 = \frac{3}{4}(x - 8).$

3. Use logarithmic differentiation to find the derivative of $f(x) = x^x$.

$$y = x^x$$

$$\frac{\mathrm{d}}{\mathrm{d}x} \ln y = \frac{\mathrm{d}}{\mathrm{d}x} \ln\bigl(x^x\bigr).$$

$$\frac{\mathrm{d}}{\mathrm{d}x} \ln y = \frac{\mathrm{d}}{\mathrm{d}x} x \ln x.$$

$$\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x} = x \cdot \frac{1}{x} + 1 \cdot \ln x.$$

$$\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x} = 1 + \ln x.$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = y(1 + \ln x).$$

$$f'(x) = x^x(1 + \ln x).$$

If you left it as $y(1 + \ln x)$, I took off a point. The question was about a function of x.