MATH 141 Fall 2022, QUIZ 4 ANSWERS

1.a. $f'(x) = 8x$

1.b. $f'(x) = 8x \cdot (3x^4 + x^3 - 7x + 1) + (4x^2 - 5) \cdot (12x^3 + 3x^2 - 7)$
This is product rule. No need to simplify further.

1.c. Rewrite $f(x) = x^{\frac{1}{3}} + 5x^{\frac{1}{2}}$, so $f'(x) = \frac{1}{3}x^{-\frac{2}{3}} + \frac{5}{2}x^{-\frac{1}{2}} = \frac{1}{3\sqrt[3]{x^2}} + \frac{5}{2\sqrt{x}}$
You could probably get away with leaving out that last simplification step...

1.d. $f'(x) = \dfrac{(x^3 - 5x + 1) \cdot (2x + 4) - (x^2 + 4x)(3x^2 - 5)}{(x^3 - 5x + 1)^2}$
This is product rule. In "real life", you would want to simplify the numerator, but on a test, we wouldn't want you to spend the time simplifying.

1.e. $f'(x) = 3\cos x + 4\sin x$

1.f. $f'(x) = x^4 \cos x + 4x^3 \sin x$

1.g. $f'(x) = \dfrac{\cos x \cdot \cos x - (\sin x + 1) \cdot (-\sin x)}{\cos^2 x} = \dfrac{\cos^2 x + \sin^2 x + \sin x}{\cos^2 x} = \dfrac{1 + \sin x}{\cos^2 x}$

There is a further simplification that you can make but you don't have to:
$$\frac{1 + \sin x}{\cos^2 x} = \frac{1 + \sin x}{1 - \sin^2 x} = \frac{1 + \sin x}{(1 - \sin x)(1 + \sin x)} = \frac{1}{1 - \sin x}$$

A cute alternative for 1.g: $f(x) = \tan x + \sec x$, *so* $f'(x) = \sec^2 x + \tan x \sec x$. *(Why is that the same?)*

1.h. $f'(x) = -\sin(x + \sqrt{x}) \cdot \left(1 + \dfrac{1}{2\sqrt{x}}\right)$

1.i.
$$\begin{aligned} f'(x) &= \cos(\sin(\sin x)) \cdot \frac{\mathrm{d}}{\mathrm{d}x}(\sin(\sin x)) \\ &= \cos(\sin(\sin x)) \cdot \cos(\sin x) \cdot \frac{\mathrm{d}}{\mathrm{d}x}\sin x \\ &= \cos(\sin(\sin x)) \cdot \cos(\sin x) \cdot \cos x \end{aligned}$$

2. *Using differentiation rules:*
$f'(x) = 2\sin x \cos x + 2\cos x(-\sin x) = 2\sin x \cos x - 2\sin x \cos x = 0$

Using the trig identity:
$f(x) = \sin^2 x + \cos^2 x = 1$, and the derivative of a constant is 0, so $f'(x) = 0$.

3. Slope of the tangent line = derivative. We compute $f'(x) = 6(x^2 - 1)^5 \cdot 2x$ by the Chain Rule. This is zero when $x^2 - 1 = 0$ (so $x = \pm 1$) or $2x = 0$ (so $x = 0$). To find y-values, we use $f(x)$ (*not* $f'(x)$). $f(-1) = 0$ and $f(0) = 1$ and $f(1) = 0$, so the three points are $(-1, 0)$, $(0, 1)$, and $(1, 0)$.