MATH 141 Fall 2025, QUIZ 3 ANSWERS *The questions on your quiz may have been slightly different.*

1. Use the Intermediate Value Theorem (IVT) to show that $f(x) = x^2 + x - 1$ has a zero between -2 and 0.

Here is a very complete explanation:
Compute $f(-2) = (-2)^2 + (-2) - 1 = 1$ and $f(0) = 0^2 + 0 - 1 = -1$. The function $f(x)$ is continuous everywhere because it is a polynomial. Since $f(-2) > 0 > f(0)$, the Intermediate Value Theorem says that there exists c with $-2 < c < 0$ such that $f(c) = 0$. In other words, $f(x)$ has a zero between -2 and 0. *The fact that $f(x)$ is **continuous** is crucial. (Look back at your homework for an example where a function **fails** to have a root because it is not continuous.)*

Here is an acceptable answer that writes much less:
$f(-2) = 1 > 0$ and $f(0) = -1 < 0$ and $f(x)$ is continuous. IVT says $f(x)$ is 0 somewhere between -2 and 0.

2. *Since you know how to do this derivative using rules, you know the answer. So I have to grade strictly, and not accept your computation if you just "made it come out" as the right answer, for the wrong reasons.*

$$f(x) = x^2 - 3x - 1$$

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{(x+h)^2 - 3(x+h) - 1 - (x^2 - 3x - 1)}{h} \\ &= \lim_{h\to 0} \frac{x^2 + 2xh + h^2 - 3x - 3h - 1 - x^2 + 3x + 1}{h} \\ &= \lim_{h\to 0} \frac{2xh + h^2 - 3h}{h} \\ &= \lim_{h\to 0} (2x + h - 3) \\ &= 2x - 3 \end{aligned}$$

If your final answer was "$\lim_{h\to 0} (2x-3)$", that wasn't correct. You already took the limit.

Please read what I said on the quiz paper at the bottom.

Why would I expect you to remember the definition of the derivative? Because there are *two conceptual reasons* why the definition of the derivative makes sense. If you are having trouble remembering the definition, ***don't try harder to memorize***. Instead, look back at the material about *instantaneous velocity* and *slope of the tangent line* and keep thinking about it and asking about it until it makes sense to you.

Just for fun, here are some more examples.

$$f(x) = 3x^2 - 2$$

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{3(x+h)^2 - 2 - (3x^2 - 2)}{h} \\ &= \lim_{h\to 0} \frac{3x^2 + 6xh + 3h^2 - 2 - 3x^2 + 2}{h} = \lim_{h\to 0} \frac{6xh + 3h^2}{h} = \lim_{h\to 0}(6x + 3h) = 6x \end{aligned}$$

$$f(x) = \frac{1}{x+1}$$

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{\frac{1}{x+h+1} - \frac{1}{x+1}}{h} \\ &= \lim_{h\to 0} \frac{1}{h} \cdot \frac{(x+1) - (x+h+1)}{(x+h+1)(x+1)} \\ &= \lim_{h\to 0} \frac{1}{h} \cdot \frac{-h}{(x+h+1)(x+1)} = \lim_{h\to 0} \frac{-1}{(x+h+1)(x+1)} = \frac{-1}{(x+1)^2} \end{aligned}$$

$$f(x) = \frac{1}{x^2}$$

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h} \\ &= \lim_{h\to 0} \frac{1}{h} \cdot \frac{x^2 - (x+h)^2}{x^2(x+h)^2} = \lim_{h\to 0} \frac{1}{h} \cdot \frac{-2xh - h^2}{x^2(x+h)^2} = \lim_{h\to 0} \frac{-2x - h}{x^2(x+h)^2} = \frac{-2x}{x^4} = \frac{-2}{x^3} \end{aligned}$$