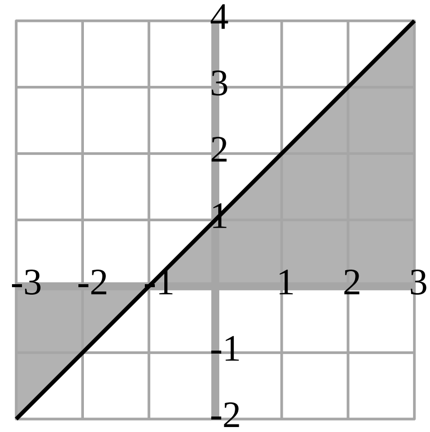
MATH 141, Fall 2025, QUIZ 12 answers

1. $\displaystyle\int_{-3}^{3}(x+1)\,\mathrm{d}x = \frac{1}{2}\cdot 4\cdot 4 - \frac{1}{2}\cdot 2\cdot 2 = 8-2=6.$

The small triangle contributes a negative area and the large triangle contributes a positive area. Note that this is not a Riemann sum problem. We're not approximating this area using rectangles, we're computing it using the fact that it is made up of triangles.

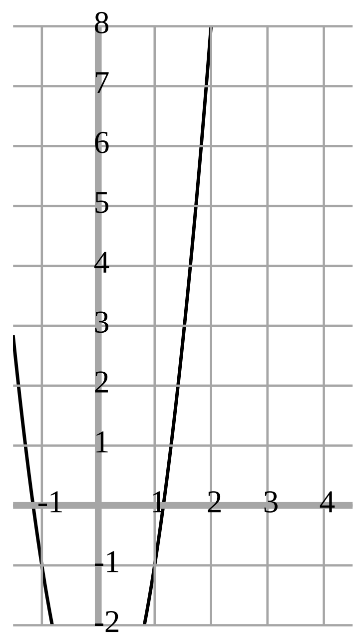


2. Given $F(x)=\displaystyle\int_0^x(\sec 2t+5t+e^t)\,\mathrm{d}t$, FTC (Part 1) says $\dfrac{\mathrm{d}F}{\mathrm{d}x}=\sec 2x+5x+e^x$.

*The answer is **not*** $\sec 2t+5t+e^t$, *because* $\frac{\mathrm{d}F}{\mathrm{d}x}$ *is a function of* x, *not* t.
*The answer is **also not*** $(\sec 2x+5x+e^x)\,\mathrm{d}x$, *because the problem asks for* $\frac{\mathrm{d}F}{\mathrm{d}x}$, *not* $\mathrm{d}F$.
Your numbers might have been slightly different.

3. $\displaystyle\int_1^2(3x^2-4)\,\mathrm{d}x = \left[x^3-4x\right]_1^2 = (2^3-4\cdot 2)-(1-4) = 0-(-3) = 3.$

This used FTC, but we won't need to keep saying that. It's just the way we'll do definite intervals. You didn't have to draw a picture, but here is one. Does the area look right?



4. $\displaystyle\int_0^{\frac{\pi}{3}}\cos x\,\mathrm{d}x = \Big[\sin x\Big]_0^{\frac{\pi}{3}} = \sin\!\left(\frac{\pi}{3}\right)-\sin(0) = \frac{\sqrt{3}}{2}-0=\frac{\sqrt{3}}{2}.$

5. $\displaystyle\int_1^3\left(e^x-\frac{1}{x}\right)\mathrm{d}x = \Big[e^x-\ln|x|\Big]_1^3 = (e^3-\ln 3)-(e^1-\ln 1) = (e^3-\ln 3)-(e-0) = e^3-\ln 3-e.$

6. Given that $\displaystyle\int_1^2 f(x)\,\mathrm{d}x = a$ and $\displaystyle\int_1^3 f(x)\,\mathrm{d}x = 1$, *(Your a was some specific number.)*

$\displaystyle\int_2^3 f(x)\,\mathrm{d}x = 1-a$

$\displaystyle\int_1^1 f(x)\,\mathrm{d}x = 0$ *This is true for any function.*

$\displaystyle\int_1^2 3f(x)\,\mathrm{d}x = 3a$