MATH 141, Fall 2025, QUIZ 10 answers.

For each of the following limits, a. identify which indeterminate form occurs and b. find the limit.

Your problem had a specific number for the "a" shown below.

1. $\displaystyle\lim_{x\to 0^+} \frac{\ln(1+x)}{ax}$.

This is a $\frac{0}{0}$ indeterminate form, because $\lim_{x\to 0^+} \ln(1+x) = \ln(1+0) = \ln 1 = 0$ and $\lim_{x\to 0^+} ax = 2\cdot 0 = 0$. We use l'Hôpital's rule, and then we can do the limit directly:

$$\lim_{x\to 0^+} \frac{\ln(1+x)}{ax} = \lim_{x\to 0^+} \frac{\frac{1}{1+x}}{a} = \frac{1}{a}$$

2. $\displaystyle\lim_{x\to 0^+} (1+x)^{\frac{1}{ax}}$.

This is a 1^∞ indeterminate form, because $\lim_{x\to 0^+}(1+x) = (1+0) = 1$ and $\lim_{x\to 0^+} \frac{1}{ax} = \infty$. We use the $e^{\ln}$ trick and continuity of e^x, and then use the answer from Problem 1:

$$\lim_{x\to 0^+} (1+x)^{\frac{1}{ax}} = \lim_{x\to 0^+} e^{\ln\left((1+x)^{\frac{1}{ax}}\right)} = \lim_{x\to 0^+} e^{\frac{1}{ax}\cdot\ln(1+x)} = e^{\lim_{x\to 0^+} \frac{\ln(1+x)}{ax}} = e^{\frac{1}{a}}.$$

Some of you want to use a different way of writing this that you've seen before, and that's fine. You don't have to write it the way I write it, but you you *can't start talking about some "y" without telling me what y is* and you still *have to make sense.* Here is a way to do what you are trying to do that makes sense. (If you don't like this, ignore it. It's not my recommended method. It makes things more confusing, IMHO.)

If you let y stand for the limit you're trying to take, then using **continuity of** $\ln x$, you can say that $\ln y = \lim_{x\to 0^+} \ln\left((1+x)^{\frac{1}{ax}}\right)$, and then use a logarithm rule: $\ln y = \lim_{x\to 0^+} \frac{1}{ax}\ln(1+x) = \lim_{x\to 0^+} \frac{\ln(1+x)}{ax}$. By Problem 1, that is $\frac{1}{a}$. Since $\ln y = \frac{1}{a}$, also $y = e^{\frac{1}{a}}$. (So all you're doing is naming something y to avoid saying $e^{\ln}$, and then you get to use continuity of $\ln x$ instead of continuity of e^x. Is that better?)

3. If the limit in Problem 1 were $x \to 0$ or $x \to 0^-$, the solution I wrote above would still work exactly as I wrote it. And this makes sense, because as $x \to 0^+$, both $\ln(1+x)$ and ax are positive, but as $x \to 0^-$, both $\ln(1+x)$ and ax are negative, so the signs cancel out.

For Problem 2, the situation is a bit more subtle: When I wrote that Problem 2 is a 1^∞ indeterminate form, I wrote $\lim_{x\to 0^+} \frac{1}{ax} = \infty$. But $\lim_{x\to 0^-} \frac{1}{ax} = -\infty$ and $\lim_{x\to 0} \frac{1}{ax}$ doesn't exist. So what I wrote to justify the answer would be subtly wrong if I were taking the limit as $x \to 0^-$ or as $x \to 0$.

On the other hand, if I wanted to do the variation of Problem 2 with $x \to 0^-$, I could do it exactly as I wrote above, except that I would write $\lim_{x\to 0^-} \frac{1}{ax} = -\infty$. (Remember, we don't care about the signs $\pm$ on ∞ when we talk about 1^∞ indeterminate forms, or the other indeterminate forms with ∞). We would still arrive at $e^{\lim_{x\to 0^-} \frac{\ln(1+x)}{ax}}$, which would still be $e^{\frac{1}{a}}$.

Since $\lim_{x\to 0^+} (1+x)^{\frac{1}{ax}} = \lim_{x\to 0^-} (1+x)^{\frac{1}{ax}} = e^{\frac{1}{a}}$, also $\lim_{x\to 0} (1+x)^{\frac{1}{ax}} = e^{\frac{1}{a}}$.