

Every problem was an indefinite integral, so the answer is only certain up to adding an arbitrary constant. The “+C” is necessary, or your answer is not correct.

$$1. \int \frac{1}{1+x^2} dx = \arctan x + C.$$

No tools. Just an antiderivative.

$$\begin{aligned} 2. \int \frac{x}{1+x^2} dx \quad & \text{substitution: } u = 1+x^2 \\ & du = 2x dx. \\ & = \frac{1}{2} \int \frac{1}{u} du = \frac{1}{2} \ln |u| + C = \frac{1}{2} \ln(1+x^2) + C \end{aligned}$$

$$\begin{aligned} 3. \int x e^x dx \quad & \text{parts: } u = x \quad v = e^x \\ & du = dx \quad dv = e^x dx \\ & = x e^x - \int e^x dx = x e^x - e^x + C \end{aligned}$$

Get into the habit of writing dx in your work as you do integration by parts! Why? Because it will help you not to make mistakes. On tests, I see some people make mistakes because they put v where dv should have been, etc. But if you have a dx in your expression for dv , it's easier to notice your mistakes, because you'll see a dx where it shouldn't be, or you'll be missing a dx , and you'll notice that.

$$\begin{aligned} 4. \int \frac{\ln x}{x^8} dx \quad & \text{parts: } u = \ln x \quad v = -\frac{1}{7x^7} \\ & du = \frac{1}{x} dx \quad dv = \frac{1}{x^8} dx \\ & = \frac{-\ln x}{7x^7} + \frac{1}{7} \int \frac{1}{x^8} dx = \frac{-\ln x}{7x^7} - \frac{1}{49x^7} + C \end{aligned}$$

Putting $dv = x^8 dx$ was not right! Since the powers of x are in the denominator, you need $dv = x^{-8} dx$.

$$\begin{aligned} 5. \int \ln x dx \quad & \text{parts: } u = \ln x \quad v = x \\ & du = \frac{1}{x} dx \quad dv = dx \\ & = x \ln x - \int 1 dx = x \ln x - x + C \end{aligned}$$