MATH 141, Mixed integral practice ANSWERS.

You might have found different ways to do some of these. (Often there are different paths to the same place.)

$$\int xe^{\frac{x}{2}}\,\mathrm{d}x = 2xe^{\frac{x}{2}} - 2\int e^{\frac{x}{2}}\,\mathrm{d}x = 2xe^{\frac{x}{2}} - 4e^{\frac{x}{2}} + C$$

$u = x \qquad v = 2e^{\frac{x}{2}}$
$\mathrm{d}u = \mathrm{d}x \quad \mathrm{d}v = e^{\frac{x}{2}}\,\mathrm{d}x$

$$\int xe^{1-x^2}\,\mathrm{d}x = -\frac{1}{2}\int e^u\,\mathrm{d}u = -\frac{1}{2}e^u + C = -\frac{1}{2}e^{1-x^2} + C$$

$u = 1 - x^2$
$\mathrm{d}u = -2x\,\mathrm{d}x$

$$\int \frac{x}{(1+x)^2}\,\mathrm{d}x = \frac{-x}{1+x} + \int \frac{1}{1+x}\,\mathrm{d}x = \frac{-x}{(1+x)^2} + \ln|1+x| + C$$

$u = x \qquad v = \frac{-1}{1+x}$
$\mathrm{d}u = \mathrm{d}x \quad \mathrm{d}v = \frac{1}{(1+x)^2}\,\mathrm{d}x$

$$\int \frac{x}{\sqrt{1-x^2}}\,\mathrm{d}x = -\frac{1}{2}\int \frac{1}{\sqrt{u}}\,\mathrm{d}u = -\frac{1}{2}\cdot\frac{2}{1}\sqrt{u} + C = -\sqrt{1-x^2} + C$$

$u = 1 - x^2$
$\mathrm{d}u = -2x\,\mathrm{d}x$

$$\int x\sin(x^2)\,\mathrm{d}x = \frac{1}{2}\int \sin u\,\mathrm{d}u = -\frac{1}{2}\cos(x^2) + C$$

$u = x^2$
$\mathrm{d}u = 2x\,\mathrm{d}x$

$$\int \frac{1}{\sqrt{1-x^2}}\,\mathrm{d}x = \arcsin x + C$$

$$\int x^2\sin x\,\mathrm{d}x = -x^2\cos x + \int 2x\cos x\,\mathrm{d}x = -x^2\cos x + 2x\sin x - \int 2\sin x\,\mathrm{d}x = -x^2\cos x + 2x\sin x + 2\cos x + C$$

$u = x^2 \qquad v = -\cos x \qquad\qquad u = 2x \qquad v = \sin x$
$\mathrm{d}u = 2x\,\mathrm{d}x \quad \mathrm{d}v = \sin x\,\mathrm{d}x \qquad\qquad \mathrm{d}u = 2\,\mathrm{d}x \quad \mathrm{d}v = \cos x\,\mathrm{d}x$

$$\int (\cos 3x - \sin x)\,\mathrm{d}x = \frac{1}{3}\sin 3x + \cos x + C$$

$$\int \frac{\ln x}{x^2}\,\mathrm{d}x = -\frac{\ln x}{x} + \int \frac{1}{x^2}\,\mathrm{d}x = -\frac{\ln x}{x} - \frac{1}{x} + C$$

$u = \ln x \qquad v = -\frac{1}{x}$
$\mathrm{d}u = \frac{1}{x}\,\mathrm{d}x \quad \mathrm{d}v = \frac{1}{x^2}\,\mathrm{d}x$

$$\int \cos^5 x\sin x\,\mathrm{d}x = -\int u^5\,\mathrm{d}u = -\frac{1}{6}u^6 + C = -\frac{1}{6}\cos^6 x + C$$

$u = \cos x$
$\mathrm{d}u = -\sin x\,\mathrm{d}x$

$$\int (3x - \ln x + e^{4x})\,\mathrm{d}x = \frac{3}{2}x^2 - x\ln x + x + \frac{1}{4}e^{4x} + C$$

Scratch Work:

$$\int \ln x\,\mathrm{d}x = x\ln x - \int x\frac{1}{x}\,\mathrm{d}v = x\ln x - \int 1\,\mathrm{d}v = x\ln x - x + C$$

$u = \ln x \qquad v = x$

$\mathrm{d}u = \frac{1}{x}\,\mathrm{d}x \quad \mathrm{d}v = \mathrm{d}x$

$$\int \arcsin x\,\mathrm{d}x = x\arcsin x - \int \frac{x}{\sqrt{1-x^2}}\,\mathrm{d}x = x\arcsin x + \frac{1}{2}\int \frac{1}{\sqrt{w}}\,\mathrm{d}w = x\arcsin x + \sqrt{1-x^2} + C$$

$u = \arcsin x \qquad v = x \qquad w = 1 - x^2$

$\mathrm{d}u = \frac{1}{\sqrt{1+x^2}}\,\mathrm{d}x \quad \mathrm{d}v = \mathrm{d}x \qquad \mathrm{d}w = -2x\,\mathrm{d}x$